classmate
Date.
Page

Note: Div, Grad, Curl all are first derivatives.

Laplacian :-

Laplacian is defined for both scalar & vector

$$\bar{\nabla}\cdot\bar{\nabla} = \nabla^2$$

$$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$$

$A_x, A_y, A_z \rightarrow$ Scalars

Cartesian :-

$$\nabla^2 T = \left(\frac{\partial^2}{\partial x^2}T + \frac{\partial^2}{\partial y^2}T + \frac{\partial^2}{\partial z^2}T\right)$$

$T \rightarrow$ Scalar

for a vector,

$$\nabla^2\vec{A} = \left(\frac{\partial^2}{\partial x^2}A_x + \frac{\partial^2}{\partial y^2}A_x + \frac{\partial^2}{\partial z^2}A_x\right)\hat{i} + \left(\frac{\partial^2}{\partial x^2}A_y + \frac{\partial^2}{\partial y^2}A_y + \frac{\partial^2}{\partial z^2}A_z\right)\hat{j}$$

$$+ \left(\frac{\partial^2}{\partial x^2}A_z + \frac{\partial^2}{\partial y^2}\cancel{A_z} + \frac{\partial^2}{\partial z^2}A_z\right)\hat{k}$$

$\boxed{\nabla^2 V = 0}$ Laplacian eqn

$\boxed{\nabla^2 V = -\rho/\epsilon_0}$ Poision eqn

Spherical Polar

$$\nabla^2 T = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial T}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(\sin\theta\frac{\partial T}{\partial\theta}\right) + \frac{1}{r^2\sin^2\theta}\left(\frac{\partial^2}{\partial\phi^2}T\right)$$

Cylendrical

$$\nabla^2 T = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial T}{\partial s}\right) + \frac{1}{s^2}\frac{\partial^2 T}{\partial\phi^2} + \frac{\partial^2 T}{\partial z^2}$$

Table for General Formula

System	u	v	w	f	g	h
C	x	y	z	1	1	1
S.P.	r	θ	ϕ	1	r	$r\sin\theta$
Cy.	s	ϕ	z	1	s	1

(i) **Gradient :-**

$$\vec{\nabla} T = \frac{1}{f}\frac{\partial T}{\partial u}\hat{u} + \frac{1}{g}\frac{\partial T}{\partial v}\hat{v} + \frac{1}{h}\frac{\partial T}{\partial w}\hat{w}$$

(ii) **Divergence :-**

$$\vec{\nabla}\cdot\vec{A} = \frac{1}{fgh}\left[\frac{\partial}{\partial u}(gh A_u) + \frac{\partial}{\partial v}(fh A_v) + \frac{\partial}{\partial w}(fg A_w)\right]$$

(iii) **Curl :-**

$$\vec{\nabla}\times\vec{A} = \frac{1}{fgh}\begin{vmatrix} f\hat{u} & g\hat{v} & h\hat{w} \\ \frac{\partial}{\partial u} & \frac{\partial}{\partial v} & \frac{\partial}{\partial w} \\ fA_u & gA_v & hA_w \end{vmatrix}$$

(iv) **Laplacian :-**

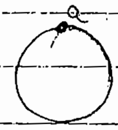
$$\nabla^2 T = \frac{1}{fgh}\left[\frac{\partial}{\partial u}\left(\frac{gh}{f}\frac{\partial T}{\partial u}\right) + \frac{\partial}{\partial v}\left(\frac{fh}{g}\frac{\partial T}{\partial v}\right) + \frac{\partial}{\partial w}\left(\frac{fg}{h}\frac{\partial T}{\partial w}\right)\right]$$

Theorems

1) <u>Gradient Theorem</u>

$$\int_a^b (\bar{\nabla} T)\cdot d\bar{r} = \int_a^b dT = T(b) - T(a)$$

$$\boxed{\oint (\bar{\nabla} T)\cdot d\bar{r} = 0}$$



bcz in close line integral initial & final points are same.

2) <u>Divergence Theorem / Gauss Theorem</u> is used to convert closed surface integral into volume integral.

$$\oint_S \bar{A}\cdot d\bar{s} = \int_V (\bar{\nabla}\cdot\bar{A})\, d\tau$$

* Close surfaces are those surfaces which bounds some volume. It can be used for sphere, cylender, cube but not for a disc bcz it is not a closed surface.

3) <u>Curl Theorem / Stoke's Theorem</u> is used to convert closed line integral into open surface integral.

$$\oint_C \vec{A}\cdot d\vec{l} = \int_S (\bar{\nabla}\times\vec{A})\cdot d\vec{s}$$

* closed line curve are those which bound some surface,

πr^2 ✓ ——— × Not a closed line curve.